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Quantum teleportation of one- and two-photon superposition states*

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Quantum teleportation of one- and two-photon superposition states based on EPR entanglement of continuous-wave two-mode squeezed state is discussed. The fidelities of teleportation are deduced for two different input quantum states. The dependence of the fidelity on the parameters of EPR entanglement and the gain of the classical channels are shown numerically. Comparing with the teleportation of Fock state and coherent state, it is pointed out that for given EPR entanglement and classical gain, the higher the nonclassicality of the input state, the lower the accessible fidelity of teleportation.

Keywords: quantum teleportation, one-photon superposition state, two-photon superposition state, fidelity

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1. Introduction

Quantum teleportation for continuous variables has been extensively studied theoretically and experimentally in recent years. Continuous-variable teleportation was first proposed by Vaidman[1] and was described in terms of the Wigner function[2−4] later. It was demonstrated by Furusawa *et al* experimentally in 1998.[5] In Ref.[6], quantum teleportation of continuous variable was formulated through the state evolution in the Schrödinger picture from the viewpoint of the general quantum mechanical measurement on the coherent state basis, and the state evolution process of teleportation was presented clearly.

So far, most theoretical discussions and performed experiments for teleportation have been focused on teleporting a coherent state. The real challenge for quantum teleportation is to teleport a quantum state, such as Fock states,[7,8] squeezed vacuum states[9] or quantum superposition state.[10]

Quantum superposition state is a kind of quantum state which plays an important role in the study of entanglement, quantum measurement, nonlocality and quantum information.[11−18] One- and two-photon superposition states are the fundamental quantum states, which have shown interesting features, such as squeezing and antibunching. People have proposed generation of superpositions of Fock state.[19] Realization of quantum superposition state is a crucial step toward designing new quantum algorithms.

The Fock state and its superposition states are the basic quantum states in quantum mechanics.[20] The simplest nonclassical superposition states are the superposition states of the vacuum states and one or two photons states.[21] Those quantum states exhibit nonclassical properties, such as squeezing and entanglement, which are essentially specific properties of nonclassical light field.

In this paper we investigate the quantum teleportation of one- and two-photon superposition states

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using entangled two-mode squeezed states, i.e. the EPR states for continuous-variable. The formula of fidelity is analytically obtained and the dependence of the fidelity on the entanglement parameter of the EPR pair[1] and the gain of the classical channels are numerically investigated. The fidelity of teleporting one-photon superposition states is higher than that of two-photon superposition states under the same conditions. Comparing with the teleportation of Fock state and coherent state, it is shown that the higher the nonclassicality of the teleported state, the lower the accessible fidelity when the available EPR entanglement and the gain of channel are given.

2. Teleportation of one-photon superposition state

The one-photon superposition state is defined[21] as

$$|\psi\rangle_{01} = C_0|0\rangle + C_1|1\rangle,$$
$$C_0 = r_0 e^{i\theta_0}, \quad C_1 = r_1 e^{i\theta_1}, \tag{1}$$

where $|0\rangle$ and $|1\rangle$ are respectively the vacuum state and the Fock state with one photon. C_0, C_1 are the corresponding coefficients, and r_0, r_1 and θ_0, θ_1 are the corresponding complex superposition probability amplitudes and phases. The normalization relation requires

$$r_0^2 + r_1^2 = 1. \tag{2}$$

According to the original teleportation scheme,[22] at the sending station, Alice holds the initial state, which is unknown for her. She shares an entangled state with Bob who is at the receiving station. For teleportation of continuous quantum variables, the entangled state distributed to Alice and Bob is a two-mode squeezed state that can be produced by parametric down-conversion in a sub-threshold optical parametric oscillator.[5,23] The density operator $\rho_{1,2}$ of the EPR entangled state can be described on the basis of two-mode Fock states:

$$\rho_{1,2} = (1-\lambda^2)\sum_{n,n'}^{\infty}(-\lambda)^{n+n'}|n,n\rangle\langle n',n'|, \tag{3}$$

where parameter $\lambda = \tanh(r)(0 \le \lambda < 1)$, and r is the squeezing factor which characterizes the entanglement of the EPR pairs.[6] The unknown state at Alice $|\psi\rangle_{01}$ is combined at a 50/50 beamsplitter with one of the EPR beam, and then the state of the combined system can be written as

$$\rho_0 = \hat{\rho}_{\rm in} \otimes \hat{\rho}_{1,2}. \tag{4}$$

Alice subsequently measures the x and y quadratures of the two output fields from the beamsplitter and this measurements provides the continuous variable analogy to Bell-state measurement for the discrete variable case. After the measurement the state collapses to the eigenstate of the quadrature amplitudes:[6]

$$\begin{aligned}\hat{\rho}_2 =& \sqrt{\frac{2(1-\lambda^2)}{\pi}}\exp[-(X^2+Y^2)|\Phi\rangle \otimes \text{h.c.},\\ |\Phi\rangle =& f(\gamma,\psi)\hat{D}[\lambda(\gamma - (\sqrt{2}g(X-iY))]|0\rangle,\\ f(\gamma,\psi) =& \frac{1}{\pi}\int_{-\infty}^{+\infty} d^2\gamma \exp\left(-\frac{1}{2}|\gamma|^2\right)\\ &\times \exp\sqrt{2}g(X-iY)\\ &\times \exp\left(-\frac{1}{2}\lambda^2\left|\gamma - (\sqrt{2}g(X-iY)\right)\right|^2,\end{aligned} \tag{5}$$

where X and Y are the results measured by Alice. Operator D is a displacement operator, which is defined as

$$\begin{aligned}&\hat{D}[\lambda(\gamma - (\sqrt{2}g(X-iY))]\\ =&\exp\{\lambda[\gamma - (\sqrt{2}g(X-iY)]a^+\\ &+\lambda[\gamma^* - (\sqrt{2}g(X+iY)]a\},\end{aligned} \tag{6}$$

a and a^+ are the annihilation and creation operators, respectively. Alice sends the classical information to Bob, and Bob uses them to perform a continuous phase space displacement on the second EPR beam and thereby generates the teleported output state

$$\begin{aligned}\rho_{\rm out} =& \sqrt{\frac{2(1-\lambda^2)}{\pi}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx dy\\ &\times \exp[-(X^2+Y^2)]\\ &\times \hat{D}[\sqrt{2}g(X-iY)]|\Phi\rangle \otimes \text{h.c.}.\end{aligned} \tag{7}$$

In Eqs.(5) and (7), $|\gamma\rangle$ is a coherent state which is a cited middle state for measurement basis and hc denotes the complex conjugate. It should be noted that, in the integration all possible measurements at Alice's station have been involved. When the input state is the one-photon superposition state $|\psi\rangle_0$, the fidelity F is deduced from the initial definition:

$$\begin{aligned}F =& \frac{1-\lambda^2}{1+g^2-2g\lambda}\left[(r_0^2 + r_1^2\lambda)^2\right.\\ &+ r_0^2 r_1^2 \frac{(g-\lambda)^2(1-g\lambda)^2}{1+g^2-2g\lambda}\\ &+ \frac{2r_1^4(g-\lambda)^2(1-g\lambda)^2}{(1+g^2-2g\lambda)^2}\\ &\left.+ \frac{r_1^2(r_0^2+r_1^2\lambda)(g-\lambda)(1-g\lambda)}{1+g^2-2g\lambda}\right],\end{aligned} \tag{8}$$

where g is the gain of the classical channel ($g \geq 0$). It is obvious that for a certain input state, the fidelity is determined by the entanglement λ and the gain of the classical channel, and is independent of the phase of the probability amplitudes θ_0, θ_1.

When r_0=1 and r_1=0, i.e., for vacuum state input, we obtain the fidelity F_0:

$$F_0 = (1-\lambda^2)/(1+g^2-2g\lambda). \tag{9}$$

If $g = 1$ and λ=0, we have F_0=1/2, the fidelity of teleporting vacuum state without entanglement.

When r_0=0 and r_1=1, we obtain the fidelity of teleporting Fock state with one photon,

$$F_1 = \frac{r_1^4(1-\lambda^2)}{1+g^2-2g\lambda}\left[\lambda^2 + \frac{2(g-\lambda)^2(1-g\lambda)^2}{(1+g^2-2g\lambda)^2} + \frac{(g-\lambda)(1-g\lambda)}{1+g^2-2g\lambda}\right]. \tag{10}$$

If g=1, λ=0, then F_1=1/4, which is the maximum fidelity of teleporting single-photon Fock state without entanglement.[24] In general, F_1 depends on g and λ, and as $\lambda \to 1$, the fidelity approaches to 1.

3. Teleportation of two-photon superposition state

The two-photon superposition state $|\psi\rangle_{02}$ is made up of a vacuum state and a two-photon Fock state,[18]

$$|\psi\rangle_{02} = C_0|0\rangle + C_2|2\rangle, \tag{11}$$

where $C_0 = r_0 e^{i\theta_0}$, $C_1 = r_2 e^{i\theta_2}$ are the complex superposition probability amplitudes. $r_0, r_1; \theta_0, \theta_1$ are their module and phases of the vacuum and two-photon state, respectively. The probability amplitudes satisfy: $r_0^2 + r_2^2 = 1$.

Similar to the above discussion, we can obtain the fidelity of teleporting two-photon superposition state $|\psi\rangle_{02}$:

$$F_2 = \frac{1-\lambda^2}{1+g^2-2g\lambda}\left[(r_0^2+r_2^2\lambda^2)^2 + \frac{4\lambda r_2^2(r_0^2+r_2^2\lambda^2)(g-\lambda)(1-g\lambda)}{1+g^2-2g\lambda} + \frac{2r_2^2(r_0^2+r_2^2\lambda^2)(g-\lambda)^2(1-g\lambda)^2 + r_0^2 r_2^2((g-\lambda)^4+(1-g\lambda)^4)}{(1+g^2-2g\lambda)^2} + \frac{8r_2^4\lambda^2(g-\lambda)^2(1-g\lambda)^2}{(1+g^2-2g\lambda)^2} + \frac{12\lambda r_2^2(g-\lambda)^3(1-g\lambda)^3}{(1+g^2-2g\lambda)^3} + \frac{6r_2^4(g-\lambda)^4(1-g\lambda)^4}{(1+g^2-2g\lambda)^4}\right]. \tag{12}$$

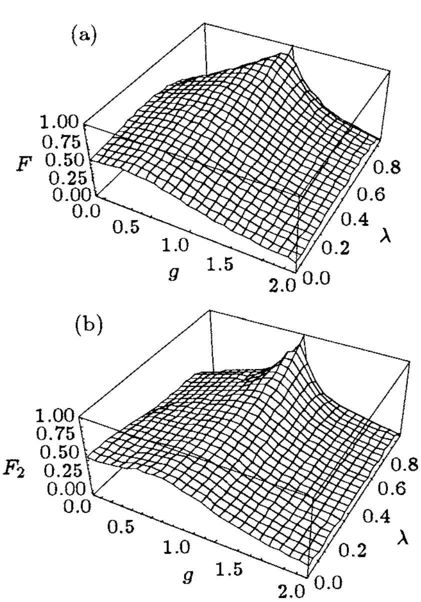

Fig.1. Fidelity F versus classical gain g and entanglement λ. We have chosen $r_0 = r_1 = r_2 = 1/\sqrt{2}$. (a) single-photon superposition state; (b) two-photon superposition state.

The fidelity of two-photon superposition state also depends on the entanglement and the classical gain. Let g=1, λ=0 and r_0=0, r_1=1, we obtain the fidelity of teleporting two-photon Fock state $|2\rangle$ without entanglement F_2=3/16.

Figure 1 shows the fidelity as a function of entanglement and gain for the above-mentioned two kinds of quantum states. The optimum gain for best fidelity is around one. The stronger the entanglement, the higher the fidelity. But compared with the one-photon superposition state, the fidelity of two-photon superposition state is more strongly dependent on the entanglement as well as the gain.

From (8) and (12) we can see that the fidelity of both one-photon superposition states and two-photon superposition states has nothing to do with the phase of complex superposition probability amplitudes C_0 and C_1 (or C_2). Consider the general superposition state with $r_0 = \sin(x)$, $r_1 = r_2 = \cos(x)$ ($0 \leq x \leq 2\pi$),

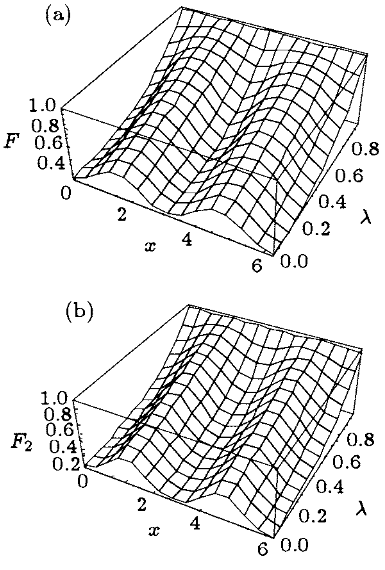

Fig.2. Fidelity F versus entanglement λ and x, Here $r_0 = \sin(x)$, $r_1 = \cos(x)$ and g=1. (a) single-photon superposition state; (b) two-photon superposition state.

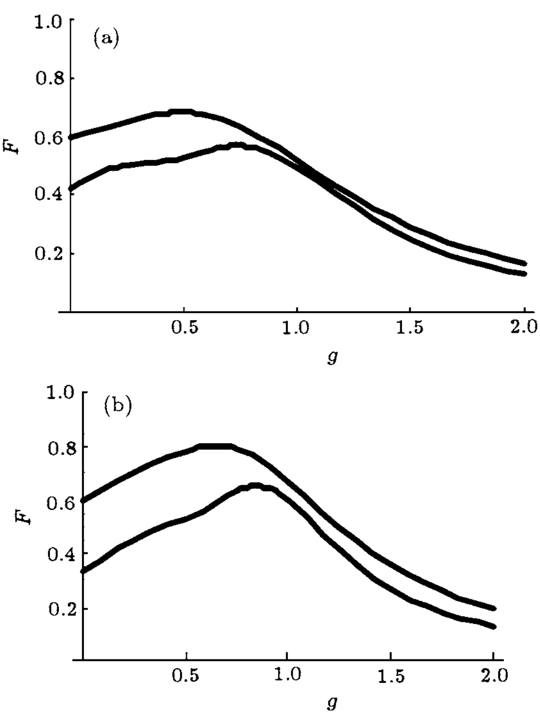

Fig.3. Fidelity F versus gain of classical channels. Here $r_0 = r_1 = r_2 = 1/\sqrt{2}$. The upper traces are for the single-photon superposition state and the lower traces for the two-photon superposition state. (a) λ=0.33; (b) λ=0.5984 (corresponding to the entanglement of 3dB and 6dB, respectively).

i.e., the module of complex superposition probability amplitudes vary from 0 to 1. Fig.2 shows the fidelity as a function of x and λ when g=1. The fidelity of the state $|\psi\rangle_{01}$ and $|\psi\rangle_{02}$ varies periodically with the probability amplitudes, which means the difficulty of teleporting an arbitrary one-photon (or two-photon) superposition states is midway between teleporting vacuum state and one-photon (or two-photon) Fock states. The stronger the entanglement, the higher the fidelity, and when the entanglement approaches to the maximum ($\lambda \to 1$) the fidelity goes to one no matter what the quantum teleported state is.

To compare the teleportation of the abovementioned quantum states, Fig.3 shows the fidelity versus the classical gain for one- and two-photon superposition states with equal superposition probability amplitudes for a given entanglement. The upper trace corresponds to the one-photon superposition states, whereas the lower trace the two-photon superposition states.

4. Conclusions

In conclusion, we have discussed the continuous-wave quantum teleportation for one- and two-photon superposition states utilizing the two-mode squeezed vacuum state to provide the needed EPR entanglement. Fidelities of teleporting those quantum superposition states are obtained. It is found that the phase of complex superposition probability amplitudes has nothing to do with the fidelity. In general the fidelity depends on the entanglement of the used EPR state and the property of the input quantum state. When the imperfect entanglement is applied the quantum state with higher nonclassical property is more difficult to be teleported. The fidelity boundaries between quantum and classical teleportation are different for different input states. For example, criteria for vacuum state, one-photon Fock state and two-photon Fock state are 1/2, 1/4 and 3/16, respectively. This result is valuable to understanding the complicate dependence of teleportation on the input quantum states.

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